



**Chairman: Nazih Abderrazzak Gadhi**

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## Welcome address

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*On behalf of the organizing committee, Sidi Mohamed Ben Abdellah University, and the LAMA laboratory, we are thrilled to welcome you to the International Conference on Optimization, ICOP 24.*


*The conference, featuring 11 keynote speakers and several oral talks, aims to gather experts in optimization and related fields from around the globe to share new ideas, spark future collaborations, and facilitate interaction. It will also provide a platform for discussing recent advances across all facets of optimization and associated disciplines. These include multi-objective optimization, set-valued optimization, mathematical programming with equilibrium or complementarity constraints, optimization under uncertainty, min-max optimization, semi-infinite optimization, bilevel optimization, integer and combinatorial optimization, robust optimization, shape optimization, optimal control, variational inequalities, and PDE-constrained optimization.*

*We wish you all a pleasant and memorable conference with open-minded and fruitful discussions.*












*Nazih Abderrazzak Gadhi*

*May, 2024*

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-  *Nazih Abderrazzak Gadhi, Sidi Mohamed Ben Abdellah University, Morocco*

## Keynote Speakers

-  *Alireza Kabgani, University of Antwerp, Belgium*
-  *El Mostafa Kalmoun, Al Akhawayn University, Morocco*
-  *Alexander Y. Kruger, Ton Duc Thang University, Vietnam*
-  *Luz de Teresa, Universidad Nacional Autonoma de México, Mexico*
-  *Juan Enrique Martinez Legaz, Universitat Autònoma de Barcelona, Spain*
-  *Fabio Raciti, University of Catania, Italy*
-  *Hassan Riahi, Cadi Ayyad University, Morocco*
-  *Vladimir Shikhman, TU Chemnitz, Germany*
-  *Michel Théra, University of Limoges, France*
-  *Alain Zemkoho, University of Southampton, United Kingdom*
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## Useful information

### Venue

The conference will take place at the faculty of sciences Dhar El Mahraz, which is part of Sidi Mohamed Ben Abdellah University in Fez, Morocco.

### Registration desk

Participants are kindly asked to pick up their conference materials at the registration desk.

### Contributed talks

Contributed talks are scheduled to last 20 minutes, including time for questions. Each lecture room is equipped with a PC. Speakers should bring their presentations on a USB flash drive and upload them onto the computer before their session begins. A member of the organizing committee will be available to assist with this process. Please arrive at least 10 minutes before your session starts.

## Special issue in Optimization

A special issue of the journal "Optimization" will be dedicated to the conference ICOP 24. All delegates are encouraged to submit a paper to contribute to this special issue by August 31st, 2024.

### Guest Editors

Nazih Abderrazzak Gadhi : [abderrazzak.gadhinazih@usmba.ac.ma](mailto:abderrazzak.gadhinazih@usmba.ac.ma)

Vladimir Shikhman: [vladimir.shikhman@mathematik.tu-chemnitz.de](mailto:vladimir.shikhman@mathematik.tu-chemnitz.de)

### Format of manuscripts

The format of manuscripts for Optimization as well as guidelines and templates can be found on the journal's web page

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Detailed instructions are provided during the submission process. When submitting their manuscript, authors must answer the question '**Is the manuscript a candidate for a special issue?**' Please tick 'yes' and select '**ICOP 24**' from the list.

## Plenary sessions

## On variational inequalities using directional convexificators

Nazih Abderrazzak Gadhi<sup>1</sup>

<sup>1</sup>Faculty of Sciences Dhar El Mahraz, Sidi Mohamed ben Abdellah University, Fez - Morocco

### Abstract

Given its pivotal role in differential calculus, the mean-value theorem has undergone numerous generalizations in the field of nonsmooth analysis over recent decades. In this work, we also look at this concept by demonstrating a mean-value theorem formulated with directional convexificators for lower semicontinuous functions. To apply our findings, we first reformulate the variational inequalities of Stampacchia and Minty in terms of directional convexificators and then directly link them to an optimization problem. Employing a specific directional generalized convexity, we then establish the necessary and sufficient conditions for a point to be an optimal solution to the optimization problem.

# High-order Moreau envelope in the nonsmooth and nonconvex setting

Alireza Kabgani<sup>1</sup>

<sup>1</sup>Department of Mathematics, University of Antwerp, Belgium

In collaboration with Masoud Ahookhosh

## Abstract

In this talk, we introduce the high-order Moreau envelope and high-order proximal operator for nonsmooth and nonconvex functions. After establishing the foundational properties of these concepts, we delve into the differentiability of the high-order Moreau envelope under prox-regularity and weak-convexity. We demonstrate that, under certain conditions, the high-order proximal operator and the gradient of our envelope satisfy the Hölder continuity. This analysis serves as the basis for the development of an inexact gradient method with a constant step-size. We illustrate the convergence properties to stationary points of both the original and envelope functions. We finally establish the linear convergence of the sequence generated by our gradient method under the Kurdyka-Łojasiewicz inequality.

# **A theoretical analysis of bilevel optimization formulations for optical flow**

El Mostafa Kalmoun<sup>1</sup>

<sup>1</sup>School of Science and Engineering, Al Akhawayn University, Ifrane - Morocco

## **Abstract**

This presentation will provide a theoretical analysis of bilevel optimization formulations for computing optical flow in a sequence of images. A general bilevel program is formulated where the lower-level problem reconstructs the flow field and the upper-level problem learns model parameters. The existence of solutions is discussed and optimality conditions are derived through a smoothing approximation in case the lower-level model problem is taken to be nonsmooth.



## Fuzzy multiplier rules: decoupling approach revisited

Alexander Kruger<sup>1</sup>

<sup>1</sup>Ton Duc Thang University, Vietnam

In collaboration with Marián Fabian and Patrick Mehlitz

### Abstract

We revisit the decoupling approach to optimality conditions and subdifferential calculus developed and discussed in [1-4] as well as some recent developments. Given extended-real-value functions and on a metric space and a subset , properties of the type are of major importance in many areas of analysis and appear (often implicitly) in many publications. The quantity in the right-hand side of the above equality is known as uniform infimum, while the property itself is often referred to as uniform lower semicontinuity. The talk is about some extensions of such properties and their consequences. In particular, we are going to show that in many cases it suffices to consider the weaker quasiuniform lower semicontinuity property [5]:

$$\inf_U (f_1 + f_2) \leq \sup_{V \in EI(U)} \liminf_{d(x_1, x_2) \rightarrow 0, x_1 \in V, x_2 \in V} (f_1(x_1) + f_2(x_2))$$

where  $EI(U)$  denotes the collection of essentially interior subsets of  $U$  :  $V \in EI(U)$  if and only if  $B_\rho(V) \subset U$  for some  $\rho > 0$ .

### References

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- [2] Borwein, J.M., Zhu, Q.J.: Viscosity solutions and viscosity subderivatives in smooth Banach spaces with applications to metric regularity. SIAM J. Control Optim. 34(5), 1568–1591 (1996).
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- [5] Fabian, M., Kruger, A. Y., Mehlitz, P.: Fuzzy multiplier, sum and intersection rules in non-Lipschitzian settings: Decoupling approach revisited. J. Math. Anal. Appl. 532(2), 127985 (2024).

## On Stackelberg hierarchic control for parabolic equations

Luz de Teresa<sup>1</sup>

<sup>1</sup>Universidad Nacional Autonoma de México, Mexico

### Abstract

In this talk we will present a Stackelberg strategy to control parabolic equations. That means we will act on the system with two controls: one will act as a leader and the other as a follower. We will discuss the different problems as Mayn objective a null controllability one and the follower been an optimization one. Then, we will present a result inverting the objectives of the leader and the follower. The leader will have an optimization target and the follower a null controllability one.

## Closed convex sets that are both Motzkin decomposable and generalized Minkowski sets

Juan Enrique Martínez-Legaz<sup>1</sup>

<sup>1</sup>Departament d'Economia i d'Història Econòmica Universitat Autònoma de Barcelona Spain

### Abstract

In this joint work with Cornel Pintea, we consider and characterize closed convex subsets of the Euclidean space that are simultaneously Motzkin decomposable and generalized Minkowski sets or, shortly, MdgM sets. We also prove the existence of suitably defined fixed points for (possibly multivalued) functions defined on MdgM sets along with existence of classical fixed points for some single valued self functions of MdgM sets. The first mentioned type of existence results are based on Kakutani fixed point theorem, and the second type are obtained by combining the Brouwer fixed point theorem with the Banach contraction principle.

# Variational inequality approach to network games: theory, algorithms and extensions

Fabio Raciti<sup>1</sup>

<sup>1</sup>Department of mathematics and computer science, University of Catania, Catania

## Abstract

Network Games (NG), or games played on networks, are a class of games where each individual (player) is identified with a node of a graph and the players that can interact directly are connected through links of the graph. The specificity of these games is the central role played by the graph structure in the description of the patterns of interactions, and in the final social or economic interpretation. The theory of Variational Inequalities (VI) has not yet been exploited in the context of NG, with only a few exceptions. In this talk we will describe the benefit of applying the VI approach to NG. In particular we derive extensions of the Bonacich-Katz representations formula to the case of bounded strategies and present an algorithm which outperforms the classical ones utilized in this field. Finally, we also present an extension to the case where the Network Game falls in the class of Nash equilibrium problems with shared constraints.

# Combining strong convergence, values fast convergence and vanishing of gradients for a proximal point algorithm using Tikhonov regularization in a Hilbert space

Hassan Riahi<sup>1</sup>

<sup>1</sup>Departement of Mathematics, Sémlalia Faculty of Sciences, Cadi Ayyad University, Marrakech, Morocco.

In collaboration with Akram Chahid Bagy, Fouad Battahi and Zaki Chbani

## Abstract

In a real Hilbertian setting, we consider a smooth convex minimization problem with nonempty solutions set. Our object is to attain strongly a suitable minimum point without strong convexity condition on this convex objective function. For this we use a combination of three techniques such as the time scaling of damped inertial gradient, the Tikhonov regularization and the Hessian-driven damping terms of such systems. We note that the time scale parameter of the damped inertial gradient improves the rate of convergences, and the term of regularization makes it possible to ensure moreover the strong convergence of iterates towards an optimal solution selected as projection of the zero vector on the set of solutions and also rapid convergence of the gradients towards zero. We note again that the Hessian-driven damping significantly reduces the oscillatory aspects with respect to times for the solutions of the proposed system. A common strategy for linear constrained minimization consists in adapting such dynamic methods for associated augmented Lagrangian saddle functions. The corresponding dynamics brings into play three general time-varying parameters, which are respectively associated with viscous damping, extrapolation and temporal scaling. By appropriately adjusting these parameters, each with specific properties, we develop a Lyapunov analysis which provides fast convergence properties of the values and of the feasibility gap. These results will naturally pave the way for developing corresponding accelerated ADMM algorithms, obtained by temporal discretization.

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## References

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- [2] A.C.Bagy, Z.Chbani, H.Riahi, Strong convergence of trajectories via inertial dynamics combining Hessiandriven damping and Tikhonov regularization for general convex minimizations. *Numerical Functional Analysis and Optimization*, 44 (2023), no. 14, pp. 1481–1509.
- [3] A.C.Bagy, Z.Chbani, H.Riahi, Fast convergence rate of values with strong convergence of trajectories via inertial dynamics with Tikhonov regularization term and asymptotically vanishing damping. To appear in *Journal of Nonlinear and Variational Analysis* Manuscript No. JNVA-2023090401
- [4] F.Battahi, Z.Chbani, H.Riahi, On the simultaneous convergence of values and trajectories of continuous inertial dynamics with Tikhonov regularization to solve convex minimization with affine constraints, Submitted to *Applied Set-Valued Analysis and Optimization*.



# Scalarization via utility functions in multi-objective optimization

Vladimir Shikhman<sup>1</sup>

<sup>1</sup>Fakultät für Mathematik, TU Chemnitz, Germany

In collaboration with Lorenzo Lampariello, Simone Sagratella and Valerio Giuseppe Sasso

## Abstract

We study a general scalarization approach via utility functions in multi-objective optimization. It consists of maximizing utility which is obtained from the objectives' bargaining with regard to a disagreement reference point. The theoretical framework for a broad class of utility functions from microeconomics is developed. For that, we associate a utility-dependent single-objective optimization problem with the given multi-objective optimization problem. We show that Pareto optimal points of the latter can be recovered by solving the former. In particular, Cobb-Douglas, Leontief, and CES utility functions are considered. We prove that any Pareto optimal point can be obtained as a solution of scalarization via one of the mentioned utility functions. Further, we propose a numerical scheme to solve utility-dependent single-objective optimization problems. Here, the main difficulty comes from the necessity to address constraints which are associated with a disagreement reference point. Our crucial observation is that the explicit treatment of these additional constraints may be avoided. This is the case if the Slater condition is satisfied and the utility function under consideration has the so-called barrier property. Under these assumptions, we prove the convergence of our scheme to Pareto optimal points. Numerical experiments on real-world financial datasets in a portfolio selection context confirm the efficiency of our scalarization approach via utility functions.

# Linear regularity and strong CHIP of closed sets in Asplund spaces, old and new results

Michel Théra<sup>1</sup>

<sup>1</sup>Mathematics and Computer Science Department, University of Limoges

## Abstract

This presentation focusses on three relevant properties for a collection of finitely many closed sets with nonempty intersection in a Banach space: linear regularity, property (G) and strong(Fréchet/limiting) CHIP and two types of strong CHIP. As well known these properties that have been previously analyzed by different authors for closed and convex sets. They play an important role in convex optimization, especially regarding constraint qualifications, error bounds and also the convex feasibility problem. For instance, in the convex setting, linear regularity is equivalent to the simultaneous fulfillment of strong CHIP and property (G), for the corresponding normal cones. This property being no longer valid outside the convex framework, it is the object of this talk to present some new results and in particular to prove the fact that Asplundity of the space may be characterized through the fact that strong limiting CHIP is a necessary condition for local linear regularity.

I will consider linear regularity for some special closed sets in convex-composite optimization. In this frame an equivalence result on linear regularity, strong Fréchet CHIP and property (G) will be presented. This result extends a dual characterization of linear regularity of finitely many closed convex sets via strong CHIP and property (G) to the non-convex case. As an application, we present a dual approach to the analysis of error bounds for inequality systems by giving several dual criteria for error bounds via Fréchet normal cones and subdifferentials.

# The bilevel optimization renaissance through machine learning: lessons and challenges

Alain Zemkoho<sup>1</sup>

<sup>1</sup>School of Mathematical Sciences, University of Southampton, UK

## Abstract

Bilevel optimization has been part of machine learning for over 4 decades now, although perhaps not always in an obvious way. The interconnection between the two topics started appearing more clearly in publications since about 20 years now, and in the last 10 years, the number of machine learning applications of bilevel optimization has literally exploded. This rise of bilevel optimization in machine learning has been highly positive, as it has come with many innovations in the theoretical and numerical perspectives in understanding and solving the problem, especially with the rebirth of the implicit function approach, which seemed to have been abandoned at some point. Overall, machine learning has set the bar very high for the whole field of bilevel optimization with regards to the development of numerical methods and the associated convergence analysis theory, as well as the introduction of efficient tools to speed up components such as derivative calculations among other things. However, it remains unclear how the techniques from the machine learning—based bilevel optimization literature can be extended to other applications of bilevel programming. For instance, many machine learning loss functions and the special problem structures enable the fulfilment of some qualification conditions that will fail for multiple other applications of bilevel optimization. In this talk, we will provide an overview of machine learning applications of bilevel optimization while giving a flavor of corresponding solution algorithms and their limitations. Furthermore, we will discuss possible paths for algorithms that can tackle more complicated machine learning applications of bilevel optimization, while also highlighting lessons that can be learned for more general bilevel programs.

## Optimal placement of sensors and actuators

Enrique Zuazua<sup>1</sup>

<sup>1</sup>Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU)

### Abstract

Most of applications addressed in the context of Applied Mathematics end up requiring controlling the process under consideration. On the other hand, efficiently controlling a system depends largely on the placement of sensors and actuators. The optimal placement problem can be formulated in many ways and often leads to complex non-convex optimization problems.

In this lecture we shall first present our earlier work in collaboration with Y. Privat and E. Trélat based in Fourier analysis. We shall discuss the wave and heat equation and describe how optimnal placements and shapes depend on the frequency range considered.

We shall later introduce the more recent work in collaboration with I. Ftouhi in which the problem is addressed from a purely geometric perspective. We also present some challenging open problems.

## Presentation Session 1

# Strong convergence of an inertial proximal algorithm with a Tikhonov regularization term for monotone inclusions.

Aïcha Balhag<sup>1</sup>

<sup>1</sup>Ecole Normale Supérieure, Marrakech, Morocco

## Abstract

In this study, our aim is to explore the convergence behaviors of sequences produced by an inertial proximal algorithm that includes a Tikhonov regularization term, generated by a maximal monotone operator  $A : \mathcal{H} \rightarrow 2^{\mathcal{H}}$ , in Hilbert spaces  $\mathcal{H}$ . Employing a temporal discretization, we adopt the methodology introduced for the analysis of continuous second-order dynamical systems by Attouch and László in 2021 [1], subsequently enhanced to incorporate Tikhonov regularization by Boţ, Csetnek, and László in 2023 [2]. Our inertial algorithm incorporates the maximally monotone operator through its Yosida approximation, with a suitable adjustment of the Yosida regularization parameter. Our analysis reveals accelerated convergence rates towards zero for both the algorithmic velocity and the Yosida regularization term. Additionally, the sequence being studied exhibits weak convergence to a zero of  $A$ , or, depending on the system's parameters, strong convergence to the minimal norm zero of  $A$ .

## References

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- [2] Radu Ioan Boţ, Ernő Robert Csetnek, Szilárd Csaba László, On the strong convergence of continuous Newton-like inertial dynamics with Tikhonov regularization for monotone inclusions, *Journal of Mathematical Analysis and Applications*, Volume 530, Issue 2, 2024.



# Proximal Block Coordinate for Constrained Nonlinear Optimization Problems

Ahmed Roubi<sup>1</sup>, Khaoula Kharfati<sup>1</sup>

<sup>1</sup>Faculty of Sciences and Technology, Settat, Morocco

## Abstract

We consider new proximal block-coordinate techniques for solving constrained continuously, not necessarily differentiable minimization problems, for which the feasible set is the Cartesian product of  $m$  closed convex sets. We propose new methods based on the method of centers and on the proximal point algorithm, for solving such problems. In these methods, at each iteration a regularized parametric problem is solved inexactly to obtain an approximation of the optimal value of the problem. The convergence of this method is guaranteed towards a stationary points, if certain assumptions are satisfied, such as the convexity and the existence of a unique minimum in each block while the other coordinates are held fixed.

## Bibliography

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## Two inertial type algorithms for monotone hierarchical equilibrium problems

Aïcha Balhag<sup>1</sup>, Zakaria Mazgouri<sup>2</sup> and Michel Théra<sup>3</sup>

<sup>1</sup>Institut de Mathématiques de Bourgogne, UMR 5584 CNRS, Université Bourgogne Franche-Comté, F-2100 Dijon, France

<sup>2</sup>National School of Applied Sciences, Sidi Mohamed Ben Abdellah University, Fez

<sup>3</sup>XLIM UMR-CNRS 7252, Université de Limoges, Limoges, France

### Abstract

The Mayn focus of this talk is on bilevel optimization on Hilbert spaces involving two monotone equilibrium bifunctions. We present a new achievement consisting on the introduction of inertial methods for solving this type of problems. Indeed, two several inertial type methods are suggested: a proximal algorithm and a forward-backward one. Under suitable conditions and without any restrictive assumption on the trajectories, the weak and strong convergence of the sequence generated by the both iterative methods are established. Two particular cases illustrating the proposed methods are thereafter discussed with respect to hierarchical minimization problems and equilibrium problems under a saddle point constraint. Furthermore, numerical examples are given to demonstrate the implementability of our algorithms. The algorithms and their convergence results improve and develop previous results in the field.

### References

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## Equilibrium Problem Resolution through a Reformulated Approach with Proximal Point Algorithm and Proximal Bundle Methods

Karima Boufi<sup>1</sup>, Abdessamad Fadil<sup>2</sup> and Ahmed Roubi<sup>2</sup>

<sup>1</sup>ISSS, Laboratoire MISI, Hassan First University of Settat, Morocco.

<sup>2</sup>FSTS, Laboratoire MISI, Hassan First University of Settat, Morocco.

### Abstract

We transform equilibrium problems into an equivalent form that is either unconstrained or less constrained. This new form incorporates the constraints into the equilibrium bifunction. Subsequently, we link this transformed representation to an optimization problem that is also unconstrained or less constrained. This connection allows us to devise a methodology for constructing general proximal bundle methods, where simpler subproblems are addressed. Our algorithm functions both as a pure proximal point algorithm and a bundle proximal method. It has the capability to generate either a feasible or infeasible sequence. Convergence to a solution of the equilibrium problem is guaranteed under the cyclically antimonotone condition imposed on the equilibrium bifunction.

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## Presentation Session 2

## Stochastic optimal control in biological wastewater treatment

Oumayma Boukhris<sup>1</sup>, Mustapha Serhani<sup>2</sup> and Abdellah Alla<sup>1</sup>

<sup>1</sup>LAMA Laboratory, Mathematics Department, Faculty of Sciences, Mohammed V University, Rabat, Morocco.

<sup>2</sup>FSJES, University Moulay IsMayl, B.P. 3102, Toulal., Meknes, Morocco.

### Abstract

In this paper, we propose a nonlinear mathematical model describing the process of biodegradation of organic pollutants by means of fungi. These fungi utilize glucose to support their metabolism and growth in the presence of white noise. Many environmental factors can impact the growth and degradation of fungi and pollutants within the system in unpredictable and random ways. Which justifies the incorporation of stochastic terms into the model. Factors such as temperature, pH levels, water composition, and light exhibit variability in ways that are difficult to anticipate, consequently influencing the biological processes occurring within the reactor. The inclusion of a stochastic term enables the consideration of these random fluctuations, leading to more precise and realistic predictions regarding the system's progression under authentic conditions. The study we're interested deals with an optimal control problem consisting in the minimization of nutrient and the maximization of dilution.

$$(S) \begin{cases} \frac{dP}{dt} = D(t)P_{in} - \frac{a_1 P}{h_P + P} F - \frac{C}{h_C + C} \frac{a_P P}{h_P + P} F - D(t)P + \sigma_P P \xi_{1,t} \\ \frac{dF}{dt} = \frac{e_1 a_1 P}{h_P + P} F + \frac{C}{h_C + C} \frac{e_2 a_P P}{h_P + P} F + f k_C \frac{C}{h_C + C} F - mF - D(t)F + \sigma_F F \xi_{2,t} \\ \frac{dC}{dt} = D(t)C_{in}^{max} u_2(t) - \frac{k_C C}{h_C + C} F + \frac{(a_C - 1) C}{h_C + C} \frac{a_P P}{h_P + P} F - D(t)C + \sigma_C C \xi_{3,t} \end{cases}$$

The cost function, related to the stochastic biological model (S), is represented as follows

$$J(x, u_1, u_2) = \mathbb{E} \left[ \int_0^T (d_1 u_1(t) + d_2 u_2(t)) dt + d_3 (P(T) + F(T)) \right].$$

The control strategy entails using the dilution rate of the reactor,  $D$ , and the glucose inflow  $C_{in}$  as controls, by modulating the inflow rate and the quantity of glucose introduced into the reactor. Mathematically speaking, two control functions,  $u_1(t) = D(t) \in [0; D]$  and  $u_2(t) \in [0; 1]$  such that  $C_{in} = C_{in}^{max} u_2(t)$ , are introduced in the model (S). This research focuses on three key aspects. Firstly, we establish the existence and uniqueness of solutions to the given problem, which serves as a fundamental step laying the groundwork for subsequent analyses and applications. Secondly, we investigate the presence of optimal control under essential conditions, a critical inquiry for determining the most efficient strategies

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to influence system dynamics. Lastly, we delve into the Hamilton-Jacobi-Bellman (HJB) equation and its viscosity solution, providing insights into system dynamics and optimal control. Integration of the HJB equation and viscosity solutions offers robustness to account for system complexities and stochastic perturbations. Our findings reveal that the HJB equation and viscosity solutions play a pivotal role in optimizing dilution rates and nutrient usage, enhancing system efficiency and pollutant biodegradation, since it provides the optimal control strategy in feedback manner, leading to possible numerical implementation. These results contribute to a comprehensive understanding of problem dynamics, with significant implications for decision-making and problem-solving scenarios.

**Keywords** Biological wastewater treatment, Stochastic optimal control, Stochastic HJB equation, Viscosity solution.

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# Dynamic Decision Modeling for Pricing policy: A Viability Approach

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## Abstract

This study aims to establish a connection between viability theory and Hamilton-Jacobi-Bellman's (HJB) theory, which is used in the framework of problem modeling of the dynamic behavior of the market. Viability theory is renowned for its application in analyzing dynamical systems' long-term behavior and durability. On the other hand, the HJB theory, a cornerstone in optimal control theory, offers a powerful mathematical tool for characterizing the optimal strategies that ensure viability while optimizing objectives.

By integrating viability theory with the HJB equation, this study aims to formulate a comprehensive mathematical model that captures the intricacies of production dynamics, considering investment in quality and advertising as integral decision variables. This fusion allows for a rigorous analysis of the production process, taking into account both short-term tactical decisions and long-term strategic considerations. Initially, the focus lies on constructing a viability kernel, describing the initial conditions from which the dynamic fulfills the viability constraints, wherein the HJB theory plays a crucial role in characterizing this kernel using viscosity solutions of an HJB equation associated with an appropriate optimal control problem. This approach gives managers insights into maintaining production and inventory levels within viable ranges, considering product quality and market demand factors.

Thereafter, the study extends its analysis to the long-term evolutionary nature of product quality, integrating it with the dynamics of market demand. From the perspective of viability theory and the HJB equation, an equilibrium between production volumes and inventory levels is sought in order to minimize the critical indicators associated with production efficiency and profitability. Furthermore, by conducting a numerical study of targeted sustainability scenarios, this research offers practical insight into decision-making processes, helping managers to formulate effective pricing policies and optimize resource allocation strategies to improve sustainability and production performance.

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# An adaptive local search-based particle swarm optimisation to solve the multi-compartment vehicle routing problem

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## Abstract

The multi-compartment vehicle routing problem MCVRP [1] is a variant of routing problems where vehicles with multiple compartments are utilized to transport different types of goods or materials simultaneously. It was widely studied in the last decades, due to its interesting applications in the real world, in domains like fuel distribution, delivery of food, garbage collection and so on. In the MCVRP, each vehicle is equipped with several compartments, each with its own capacity constraints, and the objective is to determine a set of routes for the vehicles to serve a set of customers while minimizing the total transportation cost. To effectively solve the problem, many metaheuristics were proposed in the literature, like, Hybrid Adaptive Large Neighborhood Search [4], ant colony [5], Artificial bee colony [6], and Bat Algorithm [7]. The Three last algorithms are classified as swarm intelligence algorithms, which refer to the use of collective behaviours inspired by natural swarm systems, such as ant colonies or bird flocks, to solve optimization problems [3]. In this approach, a population of potential solutions, often termed "particles" or "agents," iteratively explores the search space to find the optimal solution. Each agent adjusts its position based on both its own experience and the experiences of neighbouring agents. This paper presents a study on solving the MCVRP using Particle Swarm Optimization (PSO) [2], a nature-inspired metaheuristic algorithm known for its simplicity and effectiveness in solving optimization problems. PSO mimics the social behaviour of bird flocks or fish schools. In PSO, a population of potential solutions which defines the particles, move through the search space. Each particle adjusts its position based on its individual experience and the best-known position in the swarm, aiming to converge toward the optimal solution. We propose a PSO-based framework tailored to the MCVRP. We discuss the design considerations, including the representation of particles, initialization of the swarm, solution update mechanisms, application of local search (LS) and adaptive selection of LS operators, and termination criteria. To evaluate the effectiveness of the proposed PSO algorithm, we conduct experiments on benchmark MCVRP instances of varying sizes and complexities like those used in [5]. We compare the performance of our algorithm with other metaheuristic approaches, such as standard PSO, Bat Algorithm and ant colony optimization. Performance metrics such as solution quality and convergence rate are used for comparison. Preliminary results suggest that the adaptive local search-based PSO (ALSPSO) algorithm can efficiently find high-quality solutions to the MCVRP outperforming its standard version and

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is competitive with the state-of-the-art algorithms on certain instances. Overall, our study demonstrates the efficacy of the proposed algorithm in solving the Multicompartment Vehicle Routing Problem and highlights the significance of combining powerful mechanisms inspired by other metaheuristics into an algorithm to solve complex combinatorial optimization problems. Future work will focus on further refining the algorithm and exploring its applicability to other related optimization problems.

**Keywords** Keywords: Combinatorial Optimization, Multi-compartment Vehicle Routing Problem, Particle swarm intelligence, Metaheuristic, Optimization.

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## Optimization of the supplier selection process

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### Abstract

Due to globalization and the new characteristics of the business, companies face various challenges to ensure their continuity and competitive advantages. COVID-19 pandemic can be an extreme event that will eventually force many businesses and all industries to redesign and transform their global supply chain model? Challenges concerning Maynly reducing the operating cost which is based on selecting the optimal suppliers to provide a reliable product. This study contributes to solving a supplier selection problem under disruption risk due to the lack of literature reviews with a lack of multi-methodological perspective for the fuzzy stochastic notions and quantitative techniques for the quantification of risk alternatives. Prior studies are neglecting to consider the value of risk and prefer to discover chances for optimizing anticipated costs or profits. This study proposed a fuzzy stochastic goal programming approach for selecting the optimal supplier under disruption risk. The proposed model incorporates multiple criteria such as capacity, stochastic demand, and probability of disturbance. The problem of stochastic combinatorial optimization obtained is presented as a program of fuzzy random aim by integrating techniques of value at risk and conditional risk value. Numeric samples and calculation results are included. The results of the models help the decision-maker to optimize the selection of suppliers in the event of a disturbance risk problem by an estimated value at risk and by simultaneously minimizing the conditional value of the risk and demonstrate the efficacy and acceptability of the created risk-averse technique as well as the effects of risk factors on our model behavior.

## Presentation Session 3

# A generalized division approach for interval fractional programming problems

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## Abstract

In this work, the following interval fractional programming problem is considered with the generalized division of intervals:

The problem (IFOP) has not yet been studied in the literature. In recent years, several interesting works have been done to characterize the efficient solution of various interval optimization problems using optimality conditions and duality theory. However, in most of the works, the optimality conditions are equations, which are very restrictive and satisfy only a very small subset of solutions. Also, in almost all existing literature, the concept of differentiability of end point functions or sum of end point functions is used to characterize the differentiability of interval functions, which is a restrictive concept. The major contributions of this work are listed below:

$$(IFOP) : \min_{u \in \mathcal{U}} \phi(u) \div_g \psi(u) \\ \text{subject to } \rho_j(u) \preceq_{LU} 0, \quad \forall j \in N = \{1, 2, \dots, n\}.$$

- A generalized division approach to transforming an interval fractional problem into an equivalent nonfractional interval optimization problem is discussed.
- The necessary conditions are derived using the alternative theorem proposed and the linear independence constraint qualification.
- An alternative theorem is proposed using the empty intersection of the cone of feasible and descent directions to derive the necessary optimality conditions for the problem (IFOP).
- The LU-convexity assumption is used to prove sufficient optimality conditions and these conditions are inclusion relations rather than equations.
- A Wolfe-type dual is formulated, and the weak, strong, and strict converse duality results are derived using the LU-convexity assumption.
- A real-world application is shown in the industry sector by formulating a problem of steel blending as an interval fractional programming problem.

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## A note on $p$ –hyperconvexity in Multiobjective optimal control with orthotropic $p$ –Laplacian PDE constraints

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### Abstract

The orthotropic  $p$ –Laplacian operator [2, 3] also known as the pseudo- $p$ -Laplacian operator [1] defined as follows

$$\tilde{\Delta}_p u := \sum_{i=1}^n \frac{\partial}{\partial x_i} \left( \left| \frac{\partial u}{\partial x_i} \right|^{p-2} \frac{\partial u}{\partial x_i} \right) = \operatorname{div}(a(\nabla u(x)))$$

where  $a : \mathbb{R}^N \rightarrow \mathbb{R}^N$  with  $a(z) := \left( |z_1|^{p-2} z_1, \dots, |z_N|^{p-2} z_N \right)$  represents a significant concept in the field of mathematical analysis, particularly within the realm of partial differential equations (PDEs). It emerges as a generalization of the standard Laplacian operator to situations where the underlying material exhibits anisotropic properties, meaning that its physical characteristics vary with direction. In recent years, this operator finds extensive application in various branches of science and engineering, including materials science, biomechanics, and geophysics. Bousquet et al. in [3] proved that local weak solutions of the orthotropic  $p$ -harmonic equation are locally Lipschitz, for every  $p \geq 2$  and in every dimension. More generally, their result holds true for more degenerate equations with orthotropic structures, with right-hand sides in suitable Sobolev spaces. Chaker and Kim [4], studied the robust regularity estimates for a class of nonlinear integro-differential operators with anisotropic and singular kernels. Additionally, they proved a Sobolev-type inequality, a weak Harnack inequality, and a local Hölder estimate.

In this talk, we investigate the necessary optimality conditions for a multiobjective optimal control problem, that is governed by an elliptic orthotropic  $p$ –Laplacian equation. For that, we prove the existence and the uniqueness of a weak solution of the elliptic partial differential equation, using an appropriate energy function for which we prove both Gâteaux’s differentiability and  $p$ –hyperconvexity, with  $p \geq 2$ . Moreover, the necessary optimality conditions for a weak Pareto optimal point [5] and those for a proper Pareto optimal are established.

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# Zero-Sum Deterministic Differential Games involving continuous and Impulse Controls

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## Abstract

Motivated by applications in engineering, this study investigates new classes of two-player, zero-sum, deterministic differential games involving both continuous and impulse controls in infinite as well as finite time horizon. Impulse control problems, which characterize controlled jumps in a dynamical system, are used to model realistic features which cannot be captured by controlled differential equations. Differential games are a particular type of control problem that gives the mathematical formulation of the situations where two or more players, also called persons, are in a conflict of interest and act rationally to optimize a given pay-off. This study considers controlled ordinary differential equations where the system's state undergoes jumps at certain specific impulse times in a conflict-of-interest situation. We mainly consider three problems in the differential game and impulse control theory. We consider two-player, zero-sum, deterministic differential games where each player uses continuous and/or impulse controls. The first two problems are in infinite-time horizon and investigate only the well posedness of the considered games, where the second game problem setting allowed both players to adopt continuous control in addition to the impulsive one. In the third problem, only one player uses continuous control, and the study was extended to provide a discrete-time approximation of the value function and a Nash-equilibrium strategy for the game problem considered. First, we study a zero-sum deterministic game with two players adopting impulse controls only. Our major contribution relies on defining and studying, under rather weak assumptions on the cost functions, a new Hamilton-Jacobi-Bellman-Isaacs quasi-variational inequality for which the lower and upper value functions are the unique viscosity solution. A proportional property assumption on the maximizing player cost is assumed. The Bellman's dynamic programming principle was used to prove that the value functions are continuous and viscosity solutions to the corresponding classical Hamilton Jacobi-Bellman-Isaacs quasivariational inequality and unique solutions to the one we have defined. Second, in addition to impulse control we allow each player to use continuous control, thus a new class of two-player, zero-sum, deterministic games with discounted pay-off is formulated. The form and cost of impulses depend on nonlinear functions and on the state of the system, respectively. We reformulate the associated Hamilton-Jacobi-Bellman-Isaacs partial differential equations, we assume the Isaacs' condition, and we use the dynamic programming principle and viscosity solutions approach to show that the lower and the upper value functions coincide. Third, to

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better apply our results in the context of portfolio optimization, we have chosen to investigate a finite-time horizon, zero-sum, differential game in which the maximizing player is allowed to take continuous and impulse controls whereas the minimizing player is allowed to take impulse control only. For this class of deterministic differential games, we approximated the value function, and we provided a verification theorem. The value function was first characterized, by means of the dynamic programming principle in the viscosity solution framework, as the unique viscosity solution to the related Hamilton-Jacobi-Bellman-Isaacs doubleobstacle equation. Then an approximate value function, the unique solution to an approximate HamiltonJacobi-Bellman-Isaacs double-obstacle equation, proved to exist and converges locally uniformly towards the value function when the time discretization step goes to zero. Moreover, a Nash-equilibrium for the game problem considered was given. Finally, we derive a new continuous-time portfolio optimization model, and we provide related computational algorithms and numerical results.

**Keywords :** Impulse Control; Zero-Sum Games; Cost Function; Hamilton-Jacobi-Bellman-Isaacs Equation; Nash-Equilibrium.

## Stabilization for a class of unbounded bilinear control system in Banach space

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### Abstract

This work aims to study the problem of exponential stabilization for the following infinite dimensional bilinear system

$$\begin{cases} \frac{dx(t)}{dt} = Ax(t) + \eta(t)BCx(t), & t \geq 0 \\ x(0) = x_0 \end{cases} \quad (1)$$

Here,  $A$  is the infinitesimal generator of a linear  $C^0$ -semigroup on a Banach space  $X$  with domain  $D(A)$ . The linear operator  $B$  is bounded from a Banach space  $U$  to the extrapolation space  $X_{-1}$  of  $X$ . The linear operator  $C$  is bounded from a Banach space  $Z$  to  $X_{-1}$ . The real-valued function  $\eta(\cdot)$  denotes the control.

Using a regularizing approach, we prove the existence and uniqueness of the system's solution (1). Under sufficient conditions, we prove the stability of the solution by using an adequate bounded feedback control law. Application to the transport equation is considered.

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## Presentation Session 4

## Multi-site automotive industry: Towards optimal coordination of production, spare parts logistics and maintenance through Maintenance 4.0

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### Abstract

In light of the pivotal role of maintenance management and its impact on competitiveness in today's manufacturing landscape [1],[2], insufficient SP stock for example, can result in equipment unavailability, leading to a significant loss in case of item failures without prompt replacement. The SP policy may differ from company to another, depending on both the usage and maintenance of equipment [3]. They either store all spare parts (SP) upfront (high cost) or keep cheaper ones on hand, ordering expensive ones only when needed (risk of downtime). The automotive industry faces a double challenge: fluctuating customer demands threaten competitiveness, while inefficient production, spare parts logistics, and traditional maintenance strategies inflate costs. A unified approach leveraging Maintenance 4.0 principles for optimal spare parts management can address both issues simultaneously. Within this framework, several researchers focused their work on cost optimization for both SP reservation and maintenance or integrated production scheduling and maintenance . [4] Modeled a cost-effective production/maintenance plan in a multi-sites production system of fuel aiming to minimize the overall cost, including production, storage, shortage, transportation, and maintenance expenses.[5] Conducted his study on automotive company developing a mathematical model applied to a production scheduling aiming to improve lead time shortening, demand management and process efficiency.[6] Created an optimized integrated strategy for managing maintenance and SP for a wind turbine system, considering an observable level of degradation.[3] proposed a stochastic optimization model for both SP inventory control and the preventive maintenance (PM) inspection interval in a coordinated manner.[7] Introduced a mathematical approach utilizing probabilistic dynamic programming to optimize SP management. The emphasis is on establishing a platform that facilitates the collaborative involvement of multiple distributors in item management.[8] Implemented a cost optimization algorithm for reserving SP in transport machines by modeling the operational time of machine components until failure. [9] Developed an integrated decision

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model for production scheduling and maintenance planning aiming for a deteriorating multi-machine system (semiconductor production line) for an economic objective. The challenge involves creating an integrated production, spare parts logistics, maintenance plan within a SP management strategy that can effectively meet all specified criteria to guarantee the strive towards the minimum equipment downtime. The prior literature review indicates that numerous investigations have tackled the integration of production, maintenance planning, and other activities. Yet, there is a discernible lack of endeavors focusing on optimization of production, maintenance as well as a SP management problem in a geographically distributed multi-sites production environment especially in automotive production system field from a Maintenance 4.0 perspective. This research aims to develop an optimal production plan within a Maintenance 4.0 framework. The plan will: (1) Optimize production rate at each site (i),(2) Minimize equipment downtime caused by spare part (SP) shortages, (3) Reduce SP storage costs by SP transport coordination between sites, and finally (4) Optimize total maintenance cost. The given optimization strategy empowers sites to achieve production continuity by preventing both downtime due to spare part lead times and holding excess inventory. A numerical example will be provided to demonstrate the model's application and feasibility.

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# Efficiency Advancements in Cooperative Game Theory: The Bi-Extremal Bi-Stabilization Algorithm.

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## Abstract

Algorithmic Game Theory stands at the crossroads of computer science, economics, and game theory, delving into the exploration of strategic interactions within computational landscapes. This interdisciplinary field delves into the algorithmic dimensions of strategic decision-making and resource distribution, providing valuable perspectives on crafting effective algorithms tailored for intricate systems featuring self-interested agents. The core emphasis lies in comprehending and enhancing the results of dynamic scenarios where rational entities, be they individuals or computational agents, engage in decision-making to maximize their utility.

NASA, as one of the largest organizations, serves as a prominent example of Algorithmic Game Theory application. Specifically, cooperative game theory with transferable utilities has demonstrated efficacy, notably within NASA's PvS libraries [1]. In 2020, El Kamli et al. [2] introduced the extremal stabilization algorithm. Their initial work produced a time-efficient first-generation algorithm, followed by the development of a second-generation algorithm, consistently converging in scenarios with a non-empty core. This study extends and refines the second-generation algorithm to the cartesian product of two sets, introducing cooperative game concepts to  $N \times N'$ . The framework broadening involves a redefinition of fundamental elements, such as stability, polarity, nodes, and more, now applicable to the set  $N \times N'$ . Additionally, this work redefines the core by integrating the notion of probability.

A notable modification is the redefinition of the pivotal function  $v_{C \times C'}$  from  $2^{N \times N'}$  to  $\mathbb{R}$ , departing from the previous  $v_C$  defined exclusively from  $2^N$  to  $\mathbb{R}$  as presented in [2]. The significance of this redefined function is established, proving it to be the smallest cooperative game majorant  $v$  that is exact and features a node in  $C \times C'$ . The utilization of this function, along with the polar of a bi-stable cooperative game  $v$ , plays a crucial role in the bi-stabilization of the algorithm.

Upon juxtaposing our findings with the insights from the publication [2], we ascertain that when  $nm = |N \times N'|$ , the bi-extremal bi-stabilization algorithm for cooperative games demonstrates exceptional efficiency. To elaborate, it can resolve a system in the form of  $(S_1)$  within a mere  $2^{nm}$  iterations, marking a notable leap forward compared to the simplex algorithm. The simplex algorithm, under similar

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circumstances, typically requires around  $nm2^{nm}$  iterations. This substantial reduction in iteration count underscores the remarkable efficiency of the bi-extremal bi-stabilization algorithm in handling comparable scenarios.

$$(S_1) : \begin{cases} \text{Max}P(N \times N') \\ P(S \times S') \leq v^\times(S \times S') \quad ; \quad S \times S' \in 2^{N \times N'} \\ P \geq 0 \end{cases}$$

For minimization problems, we used the initial problem  $(S_2)$  of this problem (dual):

$$(S_2) : \begin{cases} \text{Min}P(N \times N') \\ P(S \times S') \geq v(S \times S') \quad ; \quad S \times S' \in 2^{N \times N'} \\ P \geq 0 \end{cases}$$

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# Optimal control for a tuberculosis model with contamination by the consumption of unpasteurized dairy products: mathematical modelling and numerical simulations

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## Abstract

In this study, we present an optimal control model of transmission tuberculosis which takes account the contamination by consuming unpasteurised dairy products. The aim of this study is to investigate the impact of different optimal control strategies on preventing tuberculosis disease. We have determined the equilibrium points of the model without control, whose local stability is guaranteed by Lyapunov's indirect method using the Routh-Hurwitz stability criterion. The results obtained show that, even if there are no bacteria in the environment, the probability of infection with tuberculosis remains high, and this is due to the consumption of unpasteurized dairy products. For this, we introduce some control strategies which represent the prevention through raising awareness of risks due to the consumption of unpasteurized dairy products, the removal and the pasteurization of such products, in addition to the effort of ensuring to patients treatment and monitoring in taking their pharmaceuticals. Optimal control theory is used to find the optimal way to minimise the number of infected people and unpasteurized dairy products while maintaining the cost of implementing the suggested strategies as low as possible. It is demonstrated that the model has an optimal control, and Pontryagin's maximum principle is used to characterize the control function. Numerical simulations are carried out to illustrate the effect of control implementation in reducing tuberculosis disease in the population. They indicate that the incorporation of the proposed control strategies will be a very effective measure in the controlling and management of tuberculosis disease.

**Keywords:** Tuberculosis; unpasteurised dairy products; stability analysis; optimal control; numerical simulations.

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## Supplier selection using fuzzy AHP and TOSPSIS methods : case study of Morocco

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### Abstract

Today's companies face intense competition, prompting them to consider new approaches to improving quality, reducing costs and meeting production deadlines. In this context, manufacturers need to adapt to constantly evolving market requirements and be responsive to change. Supplier selection has thus become one of the most crucial decisions for managing and improving the quality of supply chains. It involves choosing the most appropriate partners to supply the goods and services required for production. This step is of the utmost importance, as suppliers have a direct impact on product quality, production costs, operational flexibility and corporate reputation. To select suppliers effectively, companies need to consider a range of criteria. These include product quality, price, delivery times, production capacity, geographical location, supplier reputation, capacity for innovation, and compliance with ethical and environmental standards. So it's a multi-criteria decision-making problem involving both qualitative and quantitative criteria. Addressing the multi-criteria problem of supplier selection requires a formal selection process. However, there is no supplier selection standard, and it must be applied according to the situation [1]. Indeed, to solve the supplier selection problem, it is difficult to adopt a uniform strategy, given the differences in financial situation and reputation among customers, even for products of similar specifications and quality. In this work, we propose a generic selection model based on the opinions of 35 Moroccan companies. This approach will enable us to understand current supplier selection practices in Morocco and identify the criteria most relevant to this market. Using the data collected from these companies, we will be able to determine the weights of the selection criteria using the FAHP (Fuzzy Analytic Hierarchy Process ) method and choose the best supplier using the FTOPSIS (Fuzzy Technique for Order of Preference by Similarity to Ideal Solution ) method. These practical and effective recommendations will help to improve the supplier selection process in the specific context of Morocco, taking into account its particularities and the needs of companies operating in this market.

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## Presentation Session 5

# An ADMM-based heuristic algorithm for optimization problems over nonconvex second-order cone

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## Abstract

Nonconvex optimization [1, 2, 3, 4, 5] is the study of the optimization problem in which the convexity is not satisfied in at least one of the objective function or constraints. Branching, which divides the feasible region into smaller parts and solves subproblems across these parts, and convex relaxations are common solution techniques employed to handle nonconvex programs. Heuristics [5], such as randomized techniques, can be utilized to find good, workable solutions, but they do not offer lower bounds on the objective value and do not, therefore, demonstrate optimality. By heuristic, we mean that the method need not realize an optimal solution, or, in fact, even a feasible solution, if one exists. A heuristic has the advantage of being significantly quicker to carry out than an exact procedure, but on the other hand, it has the disadvantage that it is not required to find an optimal solution. The alternating direction method of multipliers (ADMM) [6, 7, 8, 9] is a well-known and classic approach in the optimization community; it was introduced by Gabay, Mercier, Glowinski, and Marrocco and dates back to the 1970s. Due to its simplicity of use and successful empirical application to a variety of situations, it has gained popularity in a short time. The ADMM has received a great deal of attention and has been broadly considered to minimize the augmented Lagrangian function for optimization problems. This technique divides the variables into several blocks based on their functions, and then, by fixing the other blocks at each iteration, and the augmented Lagrangian function is minimized with regard to each block. The approach was initially addressed as an iterative method for handling convex minimization problems using parallelization [10] and was also developed for distributed processing [11]. This algorithm shares many similarities with other well-known algorithms from literature, including Bregman's iterative algorithm [12] and Dykstra's alternating projection approach [13]. ADMM can be seen as an effort to combine the advantages of dual decomposition and augmented Lagrangian strategies for constrained optimization. For review, historical information, and references on ADMM, we refer to Boyd et al. [11] and the references contained therein. The success of the ADMM for solving convex programming problems led to its extension to nonconvex programming problems [14, 15, 16]. At first glance, applying ADMM to nonconvex programming problems would appear to be a roundly rejection of the convexity assumptions that underpin ADMM's derivation. But in fact, ADMM often proves to be a potent heuristic method, even for NP-hard nonconvex problems [17]. The idea of employing ADMM as an initiative heuristic to resolve nonconvex problems was discussed in [11, Chapter 9]. Examples of the application of ADMM to



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nonconvex problems include phase retrieval [18], matrix completion and separation [19, 20], optimal power flow [14], tensor factorization [21], conformal mapping construction, directional field correction, and color image restoration [22]. The objective function in these applications may be nonconvex, nonsmooth, or both. In this work, we are interested in applying the ADMM to solve an optimization problem over the intersection of both the nonconvex second-order cone and the nonnegative orthant cone. Projection is a fundamental concept in geometry but also has many applications outside this specialty; it is indeed an essential notion in numerous disciplines. The topic of projection is of great interest because it has a variety of implementations in both pure and applied mathematics, including optimization (see, for example, [23, 24]), statistics (see, for example, [25]), convex analysis (see, for example, [26]), and linear algebra (see, for example, [27]), etc. The nonconvex second-order cone (nonconvex SOC for short) is a nonconvex extension to the convex second-order cone, in the sense that it consists of any vector divided into two sub-vectors for which the Euclidean norm of the first sub-vector is at least as large as the Euclidean norm of the second sub-vector. In an attempt to obtain an approximate solution for optimization problems over nonconvex SOC, in this talk, we present a heuristic algorithm based on the alternating direction method of multipliers to solve them, which is the core result of our study. More specifically, the approach is built in two steps: A convex optimization problem comes first, followed by a nonconvex conic optimization. The problem in the second phase can lead to an inexact solution. Our strategy will make use of an approximate projection onto the nonconvex cone. The question of convergence remains open.

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## On the partial calmness condition for an interval-valued bilevel optimization problem

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### Abstract

In this paper, we are concerned with an interval-valued bilevel optimization problem (*BIVP*) which consists of a sequence of two interval-valued optimization problems: the feasible region of the upper-level problem (*BIVP*) is determined implicitly by the solution set of the lower-level problem (*BIVP<sub>x</sub>*).

$$(BIVP) : \begin{cases} \min_{x,y} f(x,y) = [f_L(x,y), f_U(x,y)] \\ \text{subject to: } F_i(x,y) \leq 0, \forall i \in I, y \in \mathcal{SI}(x), (x,y) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \end{cases}$$

where, for each  $x \in \mathbb{R}^{n_1}$ ,  $\mathcal{SI}(x)$  is the LU-solution set of the following parametric interval-valued optimization problem,

$$(BIVP_x) : \begin{cases} \min_y g(x,y) = [g_L(x,y), g_U(x,y)] \\ \text{subject to: } G_j(x,y) \leq 0, \forall j \in J, y \in \mathbb{R}^{n_2}. \end{cases}$$

Here,  $I := \{1, \dots, p\}$ ,  $J := \{1, \dots, q\}$ ,  $q, p, n_1 \geq 1, n_2 \geq 1$  are integers and  $f_L, f_U, g_L, g_U : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$ ,  $F_i, G_j : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$  are given functions such that

$$f_L(x,y) \leq f_U(x,y), \text{ for all } (x,y) \in \Omega,$$

where

$$\Omega := \{(x,y) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} : F_i(x,y) \leq 0, \forall i \in I, \text{ and } y \in \mathcal{SI}(x)\}$$

is the feasible set of (*BIVP*). We suppose that for each  $x \in \mathbb{R}^{n_1}$  and each  $y \in \Xi(x)$ , the lower level objective interval-function  $g$  satisfies the inequality  $g_L(x,y) \leq g_U(x,y)$ , where

$$\Xi(x) := \{y \in \mathbb{R}^{n_2} : G_j(x,y) \leq 0, \forall j \in J\}$$

is the lower-level feasible sets of (*BIVP*). Bilevel problems are a notoriously difficult class of optimization problems because of their inherent nonconvexity and nondifferentiability [?, ?, ?, ?, ?, ?]. Therefore, solving problem (*BIVP*) is not an easy task. To derive necessary optimality conditions for (*BIVP*), we propose the use of a suitable partial calmness conditions (see [?, ?, ?, ?]) for (*BIVP*), under which the nonsmooth Abadie constraint qualification can be applied. The optimality conditions are given in terms of semi-regular convexificators. Examples illustrating our findings are also provided.

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## Second order optimality conditions for a bilevel optimization problem in terms of approximate Hessians

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### Abstract

In this paper, we are concerned a nonlinear bilevel optimization problem  $(P)$ . Using the Karush-Kuhn-Tucker (KKT) conditions associated to the lower-level problem, we reformulate the initial problem  $(P)$  into a one-level problem with equality and inequality constraints  $(P^*)$ . Applying approximate Hessians introduced by Jeyakumar and Luc, we establish second order necessary and sufficient optimality conditions for  $(P^*)$ . To illustrate our findings, examples are also given.

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## On uniform null controllability of transport-diffusion equations with vanishing viscosity limit

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### Abstract

This paper aims to address an interesting open problem, posed in the paper "Singular Optimal Control for a Transport-Diffusion Equation" of Sergio Guerrero and Gilles Lebeau in 2007. The problem involves studying the null controllability cost of a transport-diffusion equation with Neumann conditions, where the diffusivity coefficient is denoted by  $\varepsilon > 0$  and the velocity by  $\mathcal{B}(x, t)$ . Our objective is twofold. First, we investigate the scenario where each velocity trajectory  $\mathcal{B}$  originating from  $\bar{\Omega}$  enters the control region in a shorter time at a fixed entry time. By employing Agmon and dissipation inequalities, and Carleman estimate in the case  $\mathcal{B}(x, t)$  is the gradient of a time-dependent scalar field, we establish that the control cost reMayns bounded for sufficiently small  $\varepsilon$  and large control time. Secondly, we explore the case where at least one trajectory fails to enter the control region and reMayns in  $\Omega$ . In this scenario, we prove that the control cost explodes exponentially when the diffusivity approaches zero and the control time is sufficiently small for general velocity.

## Presentation Session 6



## A vector $(\varphi_1, \varphi_2)$ -variational principle

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### Abstract

Let  $(\mathbb{X}, \|\cdot\|_{\mathbb{X}})$  and  $(\mathbb{Y}, \|\cdot\|_{\mathbb{Y}})$  be two Banach spaces and let  $C$  be a closed convex pointed cone of  $\mathbb{Y}$ . Let us denote  $\leq_C$  the partially vector-order associated to the cone  $C$ . A vector-valued function  $f : \mathbb{X} \rightarrow \mathbb{Y}$  is said to be  $C$ -lower semi-continuous ( $C$ -lsc) ; [1,3,10] ; at  $x_0 \in \mathbb{Y}$  if, for each neighborhood  $W$  of  $f(x_0)$ , there exists a neighborhood  $V$  of  $x_0$  in  $\mathbb{X}$  such that  $f(V) \subset W + C$ . The function  $f$  is said to be  $C$ -bounded below if there exists a  $m_0 \in \mathbb{Y}$  such that  $m_0 \leq_C f(x)$  for any  $x \in \mathbb{X}$ . Let consider a vector-valued function  $f : \mathbb{X} \rightarrow \mathbb{Y}$ ,  $C$ -lsc and  $C$ -bounded below on  $\mathbb{X}$ , and let  $\mathcal{P}$  be a class of vector-valued functions  $g : \mathbb{X} \rightarrow \mathbb{Y}$  which serves as a source of possible perturbations for  $f$ .

In the particular case  $\mathbb{Y} = \mathbb{R}$ , by variational principle we mean an assertion ensuring the existence of at least one perturbation  $g \in \mathcal{P}$  for witch the reel-valued function  $f + g$  attains its minimum on  $\mathbb{X}$ .

The first variational principle was established by Ekeland [2]. In this case, the class  $\mathcal{P}$  is just the set  $\{\varepsilon\|x - a\|; \varepsilon > 0, a \in X\}$ . If  $g$  is required to be smooth, then we speak about a smooth variational principle. In this direction Borwein-Preiss [4] proved a smooth one imposing only the existence of an equivalent smooth norm  $\|\cdot\|$ . In this case,  $\mathcal{P}$  is the set of convex combinations of translates of the square of the norm. Haydon [8] showed that there exists a Banach spaces with smooth bump function without an equivalent smooth norm (a function  $b$  is bump if it has a non empty and bounded support). So, the variational principle of Borwein-Preiss is not true in this space. So that, Deville et al [5] extended the Borwein-Preiss variational principle to spaces with smooth bump function, with  $\mathcal{P}$  equal to the family of Lipschitz smooth functions.

In [9], Maaden et all defined the notion of  $(\varphi_1, \varphi_2)$ -convex functions. They say that a real valued function  $g$  defined on a Banach space  $X$  is  $(\varphi_1, \varphi_2)$ -convex if  $g(\varphi_1(x, y, \lambda)) \leq \varphi_2(x, y, \lambda, g)$  for all  $x, y \in X$  and for all  $\lambda \in [0, 1]$ , where  $\varphi_1$  is a function from  $X \times X \times [0, 1]$  in  $X$  and  $\varphi_2$  is a function from  $X \times X \times [0, 1] \times \mathcal{F}$  in  $\mathbb{R}$ , with  $\mathcal{F}$  is a given vector space of real valued functions defined on the set  $X$ . They proved, under suitable choices of the functions  $\varphi_1$  and  $\varphi_2$ , a new variational principle for the set of  $(\varphi_1, \varphi_2)$ -convex functions. This  $(\varphi_1, \varphi_2)$ - variational principle is providing a unified framework to deduce Ekeland's, Borwein-Preiss's and Deville's variational principles.

In this work, we are interested in the vector case  $\mathbb{Y}$  ( $\mathbb{Y}$  is not necessary  $\mathbb{R}$ ) and we consider a set  $\mathcal{F}$  of vector-valued functions from  $\mathbb{X}$  in  $\mathbb{Y}$  and two vector-valued functions  $\varphi_1 : \mathbb{X} \times \mathbb{X} \times [0, 1] \rightarrow \mathbb{X}$  and

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$\varphi_2 : \mathbb{X} \times \mathbb{X} \times [0, 1] \times \mathcal{F} \longrightarrow \mathbb{Y}$  and we define the similar  $(\varphi_1, \varphi_2)$ -convex function with the partially vector-order  $\leq_C$ . So, we prove a vector form of the  $(\varphi_1, \varphi_2)$ -variational principle. In this case, the class  $\mathcal{P}$  is the set of vector-valued  $C$ -bounded below  $(\varphi_1, \varphi_2)$ -convex function  $g : \mathbb{X} \rightarrow \mathbb{Y}$  such that  $e^* \circ g(x) \rightarrow +\infty$  when  $\|x\|_{\mathbb{X}} \rightarrow +\infty$ ; where  $e^*$  is a continuous linear map. We seek to show that for  $f$  and under some properties on  $\varphi_1$  and  $\varphi_2$ , there exists a function  $g \in \mathcal{P}$  with arbitrarily small norm and the reel function  $e^*(f+g)$  attains its minimum on  $\mathbb{X}$ . More precisely, we prove that the set of all perturbations  $g \in \mathcal{P}$  such that  $e^* \circ (f+g)$  attains its strong global minimum on  $\mathbb{X}$  is residual. As corollaries, we obtain a vector version of Ekeland's, Borwein-Preiss's and Deville's variational principles shown in [6] by Finet et al.

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## Best proximity point in G-Menger spaces.

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### Abstract

In this work, we focus on nonlinear optimization problem in the context of G-Menger space. In order to obtain our objective, we introduce the best proximity point concept in G-Menger space, then we discuss the existence and uniqueness of this property. To illustrate the usability of the obtaining results, an application is given. The theorem obtained extends many results in the literature.

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## $\varepsilon$ -Weak Pareto minimality in D.C. vector optimization

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### Abstract

In this talk, we consider a vector optimization problem  $(P)$ , where the objective function and the constraint set are denoted by using differences of two vector-valued mappings, respectively. In order to get sufficient optimality conditions for an  $\varepsilon$ -weak Pareto minimal point, we introduce the  $\varepsilon$ -pseudo Diff-Max property which is a weaker version of the Diff-Max property. Using Jeyakumar's alternative theorem dedicated to cone-subconvexlike mappings, we also established necessary optimality conditions. The obtained results are given in terms  $\varepsilon$ -subdifferentials of data-functions. Examples that illustrate our findings are also given.

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# Fluidification of stochastic Petri nets by continuous Petri nets

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## Abstract

Reliability analysis often relies on stochastic models of discrete events such as Markov models and stochastic Petri nets (SPNs). However, for large dynamic systems with many components, computational complexity poses a major challenge due to the complex structure of the networks, the diversity of markings, and the specific interconnection of nodes. The analytical expression of the steady state of SPNs is complex due to the combinatorial explosion of discrete models. Furthermore, estimating average markings through simulations is time-consuming in the case of rare events. To overcome this challenge, the fluidification of Petri nets is proposed as an interesting alternative to provide a reasonable estimate of the asymptotic behavior of stochastic processes. This approach involves using a continuous approximation of SPNs, called continuous Petri nets (contPNs), to reduce computational efforts. Fluidification has been explored in various fields such as the analysis of communication protocols, manufacturing systems, and software development validation. However, this approach can be complex due to the differences between the steady states of SPNs and contPNs. This divergence emphasizes the importance of a deep understanding of these models for accurate and efficient analysis.

## Presentation Session 7

## Integer linear programming for the redundancy allocation multistate series-parallel problem

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### Abstract

We consider the problem of minimizing the linear cost of multistate homogeneous series-parallel system given the nonlinear reliability constraint on the system. We propose a simple 0-1 integer linear programming model and find optimal solutions for the test problems presented in previous research considering a constant demand corresponds to the maximum demand in the study period. The decision variables are the number of components in each subsystem, and the choice of components. The system has a finite number of performance levels varying from 0% (complete failure) to 100% (perfect function). Each level has a corresponding state probability. The system reliability is calculated using the universal generating function technique. Because of the complex nature of the problem, it is often solved by heuristics. By using an exact method, we are able to validate the solutions found by heuristics. The mathematical programming model has a relatively simple structure. It is implemented immediately with the help of a mathematical programming language and an integer linear programming software. Moreover, our method solves reasonable instances from the literature in just a few milliseconds.

The complexity of the problem depends on the application. The RAP is generally an NP-hard combinatorial optimization problem [5]. The model is complex because many factors, such as allowing mixed components or taking into account new demand levels, impact the system reliability and performance. To solve optimally the problem, we have to develop simplifying assumptions (e.g., considering constant demand, restricting each subsystem to identical components and limiting each component function to two possible states: good or failed). The components of the system are characterized by their reliability, performance, and cost; they are chosen from the relevant items available in the market. We define the system reliability to be the ability to meet the customer's performance expectation. We apply a universal moment generating function (UMGF) to evaluate the reliability [13].

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## Optimality conditions in set optimization with approximation as the generalized derivative

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### Abstract

In this paper, we are concerned with the constrained set-valued optimization problem

$$(P) : \begin{cases} \min F(x) \\ \text{subject to } G(x) \cap (-Z^+) \neq \emptyset, \ x \in S, \end{cases}$$

where  $S$  is a closed subset of  $X$ ,  $F$  and  $G$  are set-valued mappings between Banach spaces  $X$ ,  $Y$ ,  $Z$  and  $Z^+ \subset Z$  is a closed convex cone with a non empty interior.

Using the notion of set criterion introduced by Kuroiwa in [3] together with first order approximations of set-valued mappings, we give necessary optimality conditions for  $(P)$  in terms of asymptotical pointwise compact approximations. The concept of asymptotical pointwise compact approximations of a set-valued mapping takes its importance in the case of unbounded approximations. Our results are obtained without any convexity assumption. In order to get sufficient optimality conditions, first-order strong approximations of the set-valued mappings  $F$  and  $G$  are used. Since the set criterion of solution can be viewed as a weaker version of vector criterion [4], our optimality results are sharper than those of [1] and [2] where the efficient notion was used. We supply examples to illustrate advantages of our results over some recent existing ones dealing with necessary optimality conditions using the l-less order relation.

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## Sufficient optimality conditions and duality results for a fractional multiobjective optimization problem in terms of convexificators

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### Abstract

In this work, we are concerned with a fractional multiobjective optimization problem  $(P)$  involving set-valued maps. Based on necessary optimality conditions given by Gadhi, Rahou et al. [5], using support functions, we derive sufficient optimality conditions for  $(P)$ , and we establish various duality results by associating the given problem with its Mond-Weir dual problem  $(D)$ . The main tools we exploit are convexificators and generalized convexities. Examples that illustrates our findings are also given.

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## Bi-objective model for tactical planning in corn supply chain considering CO<sub>2</sub> balance

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### Abstract

The corn supply chain is one of the most important to the world economy, as this cereal is the third most cultivated worldwide, ranking second to only soybeans and wheat [?]. Corn is consumed in different ways by humans, ranging from grain itself to animal feeding to produce meat, eggs, and milk for several industrialized products, such as oils, cosmetics, polymers and ethanol. In particular, ethanol from corn is an interesting energy source that has been gaining market space and growing worldwide, mainly because it is a renewable and cleaner energy source compared to fossil fuels. Due to its relevance in food security, both for humans and animals and in various industrial applications, corn cultivation areas have been increasing in recent years. Data available in the reports on [?] showed that from year 2017 to 2022, the cultivated area increased from 193 to 201 million hectares, i.e. 4%. According to the same source, this trend has been seen in the world's corn production, rising from 1,080 billion tons in 2017 to 1,155 in 2022, and 1,220 in 2023. The most significant countries responsible for producing this grain are the United States, China and Brazil, corresponding to more than 60% of the whole world's production. According to [?], this agricultural chain should explore opportunities to take advantage of the complex means of production, infrastructure, transportation, storage, distribution, and the consumer market.

In this perspective, this work examines the Brazilian corn production logistic supply chain with an optimization model taking into account carbon emissions from transportation and carbon sequestration from corn production. The emphasis lies on the higher-level decision planning, encompassing optimal regions for corn cultivation, silo placement and sizing for grain storage, transportation modes integrating intermodality, while considering various resource constraints to fulfill both domestic market and export demands. Two conflicting objectives are under consideration: the logistics costs of the supply chain and the CO<sub>2</sub> balance. Precise data are employed to assess the model, and Pareto optimal solutions are generated within scenarios potentially applicable in real-world settings.

In view of the most related studies in the corn supply chain ([?] and [?]), this study contributes to the field and practice of operations research/optimization by introducing the following aspects:

- The proposition of a bi-objective MILP model for the corn supply chain with its specificities which consider concomitantly: (i) decisions on the agricultural and industrial sectors of the dry corn;

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(ii) multiple transportation modes in logistics to meet the demand; (iii) incidence of taxes due to product transportation; (iv) investigation into carbon capture by cultivating areas, carbon emissions due to the transportation of the production and how these aspects influence the costs in the supply chain.

- Detailed analysis of the trade-offs between carbon emission and logistic cost using the Pareto optimal solutions through several scenarios, providing managerial insights for decision-making, particularly regarding the logistic infrastructure bottlenecks.

To obtain Pareto optimal solutions, we used the AUGMENCON method, an improved version of the well-known  $\epsilon$ -constrained method. This study considered a layer graph among the various parameters, whose nodes and different connections between them translate the Brazilian grain transportation logistics.

The solutions provided pertinent insights to inform decisions regarding investment in logistic infrastructure. The model solutions revealed that the chain costs are minimized when corn planting is mostly in Brazil's South and Southeast regions, which are closer to the Mayn seaports for export, reducing logistics and fiscal efforts. This solution results in a higher corn productivity strategy, higher carbon emission rates due to the slightly more intensive use of the road, and a lower sequestration rate of CO<sub>2</sub> in the cultivation areas. This causes the carbon balance of the chain to be almost 400 thousand tons in the atmosphere, using 46% of the rail transport. On the other hand, the most sustainable solution possible is 31% more expensive but stores more than 2.6 million tons of carbon in the soil. This is caused because the corn production is shifted Maynly to the Midwest region of Brazil, raising the logistics and fiscal costs, even though the rail modal is responsible for 53% of the flow.

Investments in expanding public cooperative silos, strategically installed near the current railroads and waterways, more sustainable and economical modalities, can reduce costs by 5% and the carbon balance by more than 13% due to lower emissions. Projects to insert more railroads and increase their transport capacity can also be considered, but with smaller gains in both objectives. The results also indicated that using more railroads and waterways than the equilibrium levels in their current configuration can make the routes longer and the exploitation of road connections worse for both objectives.

This analysis adds to the government entities that silos and seaports should receive more investments to increase storage and handling capacity. More specifically, from the results, it is possible to detect which transportation modalities deserve more public investments in the short and long term to improve their connections with seaports.

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